Assessment Schedule – 2024

Scholarship Calculus (93202)

Evidence Statement

<table>
<tr><th>Q</th><th>Solution</th></tr>
<tr><td>ONE
(a)(i)</td><td>

$f(x)=\dfrac{2x^2+1}{3x^2-4x-2}$

$f'(x)=\dfrac{4x(3x^2-4x-2)-(6x-4)(2x^2+1)}{(3x^2-4x-2)^2}=\dfrac{-2(4x-1)(x+2)}{(3x^2-4x-2)^2}$

Stationary points when $x=\frac{1}{4}$ and $x=-2$.

The y-values are $-\frac{2}{5}$ and $\frac{1}{2}$ respectively.

Using the 1DT to determine their nature:

x	–3	–2	0	¼	1
y	–	0	+	0	–

So, the point $\left(-2,\frac{1}{2}\right)$ is a local minimum and $\left(\frac{1}{4},-\frac{2}{5}\right)$ is a local maximum.

</td></tr>
<tr><td>(ii)</td><td>

$\lim\limits_{x\to\pm\infty}\dfrac{2x^2+1}{3x^2-4x-2}=\dfrac{2}{3}$

Now we can find where the function equals this value:

$\dfrac{2x^2+1}{3x^2-4x-2}=\dfrac{2}{3}$

$6x^2+3=6x^2-8x-4$

$x=-\dfrac{7}{8}$

The function crosses its horizontal asymptote at $\left(-\frac{7}{8},\frac{2}{3}\right)$.

</td></tr>
<tr><td>(b)</td><td>

$x^3+y^3=2 \quad x+y=a$

$x^3+(a-x)^3=2$

$x^3+a^3-3a^2x+3ax^2-x^3-2=0$

$3ax^2-3a^2x+(a^3-2)=0$

$\Delta=(-3a^2)^2-4(3a)(a^3-2)=-3a^4+24a=-3a(a^3-8)<0$

$-3a(a-2)(a^2+2a+4)<0$

$a(a-2)((a+1)^2+3)>0$

$a(a-2)>0$

$a<0$ or $a>2$

Note: $a = 0$ is a special case that needs to be considered separately due to the equation in Line 4 not being a quadratic in this case. Upon checking, $a = 0$ does not yield any solutions either.

$\therefore$ The real values for which the system has no real solutions are: $a \leq 0$ or $a > 2$.

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</table>

(c)

$y = kx^2(126-x)$

$y' = k(2x(126-x)-x^2) = 0$

$252x - 3x^2 = 0$

At $x = 84$, we have a maximum height of 48 m.

$48 = k(84)^2(126-84) \Rightarrow k = \dfrac{1}{6174}$

Each step is 0.28m in depth, so the height of each step Δy is expressed by:

$$\Delta y = \frac{1}{6174}(x+0.28)^2(126-(x+0.28)) - \frac{1}{1674}x^2(126-x)$$

$$\Delta y = \frac{1}{6174}(-0.84x^2 + 70.3248x + 9.856448)$$

$$\frac{\mathrm{d}\Delta y}{\mathrm{d}x} = \frac{1}{6174}(-1.68x + 70.3248) = 0$$

$x = 41.86$

Checking step heights at either side:

$$\Delta y = \frac{1}{6174}(-0.84(41.72)^2 + 70.3248(41.72) + 9.856448) = 0.24 \text{ m}$$

$$\Delta y = \frac{1}{6174}(-0.84(42)^2 + 70.3248(42) + 9.856448) = 0.24 \text{ m also}$$

The stairs do not satisfy the maximum height regulations, as the biggest steps are 0.24 m high.

Alternative solutions

The location of the point of inflection can be found as it lines up with the steepest part of the hill.

$$y' = k(252x - 3x^2) > \frac{19}{28}$$

$$42 - \frac{\sqrt{7056k^2 - \frac{19k}{21}}}{2k} < x < 42 + \frac{\sqrt{7056k^2 - \frac{19k}{21}}}{2k}$$

This represents the range of x where the hill is steeper than $\frac{19}{28}$ (rather than the "slope" of an individual step).

This reduces to $22.83 < x < 61.17$ when $k = \frac{1}{6174}$ and shows there would be numerous steps that are too high.

NOTE:

If each step is built with a depth of 0.28m, then we have 300 steps $\left(\frac{84}{0.28}\right)$.

If all steps are built with a maximum height of 0.19 m, the steps would reach a height of 57 m (0.19×300), suggesting that, on average, the steps are within the height requirements. But this does not tell us whether the steps around the point of inflection are less than 0.19 m in height.

<table>
<tr><th>Scholarship level</th><th>Outstanding level</th></tr>
<tr><td>Q1(a)(i)
• 1s mark for x-values of stationary points
• 1s mark for coordinates, their nature, and justification

Q1(a)(ii)
• 1s mark for calculating limit of 2/3
• 1s mark for correct coordinates

Q1(b)
• 1s mark for Line 4 quadratic expression
• 1s mark for inequality $a > 2$

Q1(c)
• 1s mark for expression for Δy(accept if in terms of k)
• 1s mark for maximum at $x = 41.86$

8 marks available
Max 6 marks awarded</td><td>Q1(b)
• 1o mark for $a \leq 0$, $a > 2$ or $a \neq 0$ stated.

Q1(c)
• 1o mark for height checked either side (accept with k)
• 1o mark for a valid conclusion (or a final condition / inequality written in terms of k)

3 marks available
Max 2 marks awarded</td></tr>
</table>

TWO (a)	$\int_0^3 h(x)\,\mathrm{d}x = \int_0^1 2\sqrt{x}\,\mathrm{d}x + \int_1^2 2x\,\mathrm{d}x + \int_2^3 x^2\,\mathrm{d}x$ $= \left[\frac{4}{3}\sqrt{x^3}\right]_0^1 + \left[x^2\right]_1^2 + \left[\frac{x^3}{3}\right]_2^3$ $= \frac{4}{3} + 4 - 1 + 9 - \frac{8}{3} = \frac{32}{3}$
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + y^2$ Using implicit differentiation: $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2y\frac{\mathrm{d}y}{\mathrm{d}x}$ $= 2y(1 + y2) = 2y + 2y3$ $\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = 2\frac{\mathrm{d}y}{\mathrm{d}x} + 6y^2\frac{\mathrm{d}y}{\mathrm{d}x} = 2(1+3y^2)(1+y^2)$ **Alternative solutions** Finding the third derivative using the product rule: $\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = 2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + 2y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ $= 2(1+y^2)^2 + 2y\left(2y\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ $= 2(1+y^2)^2 + 2y\left(2y\left(1+y^2\right)\right)$ $= \left(2\left(1+y^2\right) + 4y^2\right)\left(1+y^2\right)$ $= 2(1+3y^2)(1+y^2)$ Hence $M = 2$ OR $\int \frac{1}{1+y^2}\,\mathrm{d}y = \int 1\,\mathrm{d}x$ $\tan^{-1}(y) = x + c$ $y = \tan(x+c)$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \sec^2(x+c)$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2\sec^2(x+c)\tan(x+c)$ $\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = 2\left(2\sec^2\left(x+c\right)\tan^2\left(x+c\right) + \sec^4\left(x+c\right)\right)$ $= 2\left(\sec^2\left(x+c\right)\left(2\tan^2\left(x+c\right) + \sec^2\left(x+c\right)\right)\right)$ $= 2\left(\sec^2\left(x+c\right)\left(3\sec^2\left(x+c\right) - 2\right)\right)$ $= 2(1+y^2)\left(3\left(1+y^2\right) - 2\right)$ $= 2(1+y^2)(1+3y^2)$

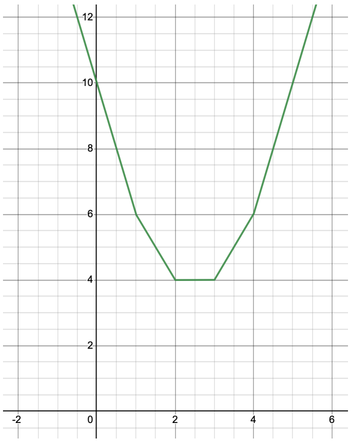
(c)(i)

For $2 \leq x \leq 3, s_3(x) = (x-1)+(x-2)-(x-3) = x$

$s_3'\left(\frac{5}{2}\right) = 1$

(ii)

The graph of $s_n(x)$ is symmetric for even values of n. An example is shown for $s_4(x)$:



The minimum value of $s_{2024}(x)$ occurs between $x = 1012$ and $x = 1013$.

$$s_{2024}(1012) = |1012-1|+|1012-2|+\ldots+|1012-2024|$$
$$= 1011+1010+\ldots+1+0+1+\ldots+1012$$
$$= 2\times\frac{1011\times1012}{2}+1012 = 1012^2$$

Alternative solution

Finding a pattern in the minimum values of the function for even values of n:

n	s_n(x)	Minimum
3	$\lvert x-1\rvert + \lvert x-2\rvert + \lvert x-3\rvert$	$s_3(2) = 2$
4	$\lvert x-1\rvert + \lvert x-2\rvert + \lvert x-3\rvert + \lvert x-4\rvert$	$s_4(2) = s_4(3) = 4 = 2^2$
5	$\lvert x-1\rvert + \lvert x-2\rvert + \lvert x-3\rvert + \lvert x-4\rvert + \lvert x-5\rvert$	$s_5(3) = 6$
6	$\lvert x-1\rvert + \lvert x-2\rvert + \lvert x-3\rvert + \lvert x-4\rvert + \lvert x-5\rvert + \lvert x-6\rvert$	$s_6(3) = s_6(4) = 9 = 3^2$
2024	$\lvert x-1\rvert + \lvert x-2\rvert + \ldots + \lvert x-2024\rvert$	$s_{2024}(1012) = 1012^2$

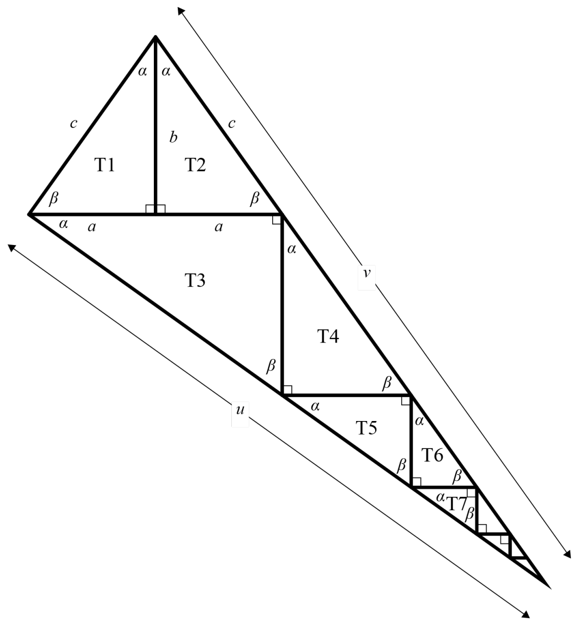
Scholarship level	**Outstanding level**
Q2(a) • **1s mark** for correct limits (only $x = 1$ and $x = 2$ needed) • **1s mark** for correct application of max function (Line 1) • **1s mark** for correct answer, with integration shown Q2(b) • **1s mark** for any correct 2nd derivative expression **1s mark** for third derivative in terms of 1 variable (x or y) Q2(c)(i) • **1s mark** for recognising $s_3(x) = x$ in required interval • **1s mark** for correct derivative Q2(c)(ii) **1s mark** for stating minimum at $x = 1012.5$ or 1012 / 1013 *8 marks available* **Max 6 marks awarded**	Q2(b) • **1o mark** for correct value of M, with working Q2(c)(ii) • **1o mark** for correct summation with no absolute values remaining (Line 3) • **1o mark** for correct minimum value, with working *3 marks available* **Max 2 marks awarded**

THREE (a)	$\int_0^{\frac{\pi}{4}} \frac{\cos 2x}{\cos^2 x}\,\mathrm{d}x = \int_0^{\frac{\pi}{4}} \frac{2\cos^2 x - 1}{\cos^2 x}\,\mathrm{d}x = \int_0^{\frac{\pi}{4}} \left(2 - \sec^2 x\right)\mathrm{d}x$ $= \left[2x - \tan x\right]_0^{\frac{\pi}{4}} = \frac{\pi}{2} - 1$
(b)(i)	$x = 4\cos\theta + \cos 4\theta$ $y = 4\sin\theta + \sin 4\theta$ $D = \sqrt{x^2 + y^2} - R$, where R is the radius of the teapot. $D = \sqrt{(4\cos\theta + \cos 4\theta)^2 + (4\sin\theta + \sin 4\theta)^2} - R$ $D = \sqrt{17 + 8\cos 4\theta\cos\theta + 8\sin 4\theta\sin\theta} - R$ $D = \sqrt{17 + 8\cos 3\theta} - R$ Minimum distance occurs when $\cos 3\theta = -1$ or by solving: $\frac{dD}{d\theta} = -\frac{24\sin 3\theta}{2\sqrt{17 + \cos 3\theta}} = 0$ $\theta = \frac{\pi}{3}$ gives the desired coordinate: $\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$ **Alternative solution** Using differentiation to find the points where the curve is not smooth (gradient is undefined): $\frac{dy}{dx} = \frac{4\cos\theta + 4\cos 4\theta}{-4\sin\theta - 4\sin 4\theta}$ $-4\sin\theta - 4\sin 4\theta = 0$ $\sin 4\theta + \sin\theta = 0$ $2\sin\frac{5\theta}{2}\cos\frac{3\theta}{2} = 0$ The values of θ for which the 1st factor equals zero are $0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}, 2\pi$. These values are where the tangent to the curve is vertical (not the desired points). The values of θ for which the 2nd factor equals zero are $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$ where $\theta = \frac{\pi}{3}$ is in the first quadrant. NOTE: If candidates set $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 0$ and then stumble across $\theta = \frac{\pi}{3}$, this is insufficient working.

(ii)	$L = 6\int_0^{\frac{\pi}{3}} \sqrt{(-4\sin\theta - 4\sin 4\theta)^2 + (4\cos\theta + 4\cos 4\theta)^2}\ \mathrm{d}\theta$ $= 6\int_0^{\frac{\pi}{3}} \sqrt{16\sin^2\theta + 32\sin\theta\sin 4\theta + 16\sin^2 4\theta + 16\cos^2\theta + 32\cos\theta\cos 4\theta + 16\cos^2 4\theta}\ \mathrm{d}\theta$ $= 6\int_0^{\frac{\pi}{3}} \sqrt{32 + 16\cos 3\theta - 16\cos 5\theta + 16\cos 5\theta + 16\cos 3\theta}\ \mathrm{d}\theta$ $= 6\int_0^{\frac{\pi}{3}} \sqrt{32 + 32\cos 3\theta}\ \mathrm{d}\theta$ $= 6\int_0^{\frac{\pi}{3}} \sqrt{64\cos^2\left(\frac{3\theta}{2}\right)}\ \mathrm{d}\theta$ $= 6\int_0^{\frac{\pi}{3}} 8\cos\left(\frac{3\theta}{2}\right)\mathrm{d}\theta$ $= \left[32\sin\left(\frac{3\theta}{2}\right)\right]_0^{\frac{\pi}{3}} = 32$ Note: If integrating from 0 to 2π i.e. $L = \int_0^{2\pi} \sqrt{32 + 32\cos 3\theta}\ \mathrm{d}\theta$ then line 6 would need to be: $L = \int_0^{2\pi} \left\lvert 8\cos\frac{3\theta}{2} \right\rvert \mathrm{d}\theta$ If the absolute value function is not used here, then the resulting integral yields 0!
(c)(i)	Using the sine rule, and cross multiplication gives us the desired result: $\frac{\sin 2\alpha}{2a} = \frac{\sin\beta}{c} = \frac{\left(\frac{b}{c}\right)}{c}$ OR $\sin 2\alpha = 2\sin\alpha\cos\alpha = 2 \times \frac{a}{c} \times \frac{b}{c} = \frac{2ab}{c^2}$ Alternative solution $A = \frac{1}{2} \times c^2 \times \sin 2\alpha = \frac{1}{2} \times 2a \times b \Rightarrow c^2 = \frac{2ab}{\sin 2\alpha}$

(ii) The waffle cone consists of an infinite number of similar right-angled triangles.

Triangle	Side adjacent to α	Side opposite to α	Hypotenuse
T1	b	a	c
T2	b	a	c
T3	$2a$	$\frac{2a^2}{b}$	$\frac{2ac}{b}$
T4	$\frac{2a^2}{b}$	$\frac{2a^3}{b^2}$	$\frac{2a^2c}{b^2}$
T5	$\frac{2a^3}{b^2}$	$\frac{2a^4}{b^3}$	$\frac{2a^3c}{b^3}$
T6	$\frac{2a^4}{b^3}$	$\frac{2a^5}{b^4}$	$\frac{2a^4c}{b^4}$
T7	$\frac{2a^5}{b^4}$	$\frac{2a^6}{b^5}$	$\frac{2a^5c}{b^5}$



$$u=\frac{2ac}{b}+\frac{2a^3c}{b^3}+\frac{2a^5c}{b^5}+\ldots$$

This is an infinite geometric series with first term $\frac{2ac}{b}$ and ratio $\frac{a^2}{b^2}$

$$u=\frac{2ac}{b}\times\frac{1}{1-\left(\frac{a^2}{b^2}\right)}=\frac{2ac}{b}\times\frac{b^2}{b^2-a^2}=\frac{2abc}{b^2-a^2}$$

$$v=c+\frac{2a^2c}{b^2}+\frac{2a^4c}{b^4}+\ldots$$

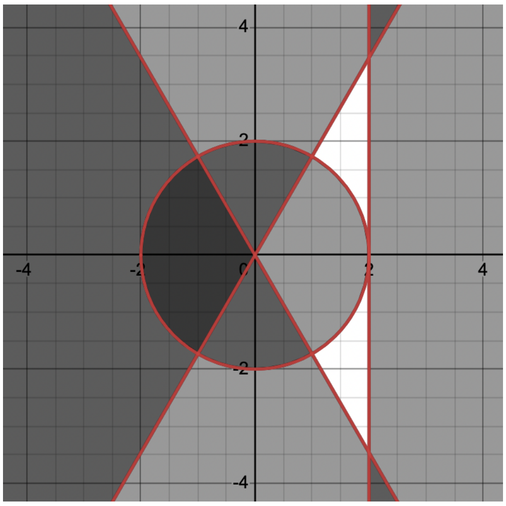
$$v=c+\frac{2a^2c}{b^2}\times\frac{1}{1-\left(\frac{a^2}{b^2}\right)}=c+\frac{2a^2c}{b^2-a^2}=\frac{cb^2+a^2c}{b^2-a^2}$$

$$\sin 2\alpha=\frac{u}{v}=\frac{2abc}{cb^2+a^2c}=\frac{2ab}{b^2+a^2}=\frac{2ab}{c^2}$$

$a^2+b^2=c^2$ as required.

	Alternative solution Using the double angle formula (this can be proven without the use of Pythagoras). $\tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha} = \dfrac{2\left(\dfrac{a}{b}\right)}{1-\left(\dfrac{a}{b}\right)^2} = \dfrac{2ab}{b^2-a^2} = \dfrac{u}{c}$ $\therefore u = \dfrac{2abc}{b^2-a^2}$ $\cos 2\alpha = 2\cos^2\alpha - 1 = 2\left(\dfrac{b}{c}\right)^2 - 1 = \dfrac{2b^2-c^2}{c^2} = \dfrac{c}{v}$ $\therefore v = \dfrac{c^3}{2b^2-c^2}$ $\sin 2\alpha = \dfrac{u}{v} = \dfrac{\dfrac{2abc}{b^2-a^2}}{\dfrac{c^3}{2b^2-c^2}} = \dfrac{2ab(2b^2-c^2)}{c^2(b^2-a^2)} = \dfrac{2ab}{c^2}$ This is true if $2b^2-c^2 = b^2-a^2$ $a^2+b^2=c^2$

Scholarship level	Outstanding level
Q3(a) • **1s mark** for simplification of integrand to $2 - \sec^2 x$ • **1s mark** for correct exact value, with integration Q3(b)(i) • **1s mark** for $17 + 8\cos(3\theta)$ or setting $\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = 0$ • **1s mark** for correct coordinates, with justification • Q3(b)(ii) • **1s mark** for correct setup of integral with limits (Line 1) • **1s mark** for perfect square simplification in Line 5 Q3(c)(i) • **1s mark** for any valid proof Q3(c)(ii) • **1s mark** for showing the expression for u *8 marks available* **Max 6 marks awarded**	Q3(b)(ii) • **1o mark** for correct answer, with supporting integration Q3(c)(ii) • **1o mark** for finding the expression for v • **1o mark** for proving the final result $a^2 + b^2 = c2$ *3 marks available* **Max 2 marks awarded**

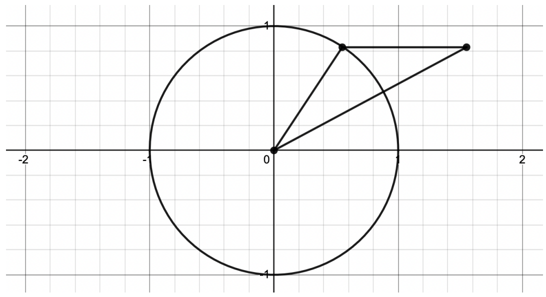
FOUR (a)	Condition One: $-\sqrt{3}x \le y \le \sqrt{3}x$ Condition Two: $x \le 2$ Condition Three: $x^2 + y^2 \ge 4$ The locus consists of the two white regions shown below (shading out)  The area of the region can be found by removing a sector from a triangle. $A = \frac{1}{2} \times 2 \times 4\sqrt{3} - \frac{1}{3} \times \pi \times 2^2 = 4\sqrt{3} - \frac{4\pi}{3}$
(b)(i)	$\text{LHS} = \frac{1}{1 - z\cos\theta}$ $= \frac{1}{1 - \cos^2\theta - \mathrm{i}\sin\theta\cos\theta}$ $= \frac{1}{\sin^2\theta - \mathrm{i}\sin\theta\cos\theta}$ $= \frac{1}{\sin\theta(\sin\theta - i\cos\theta)} \times \frac{\sin\theta + \mathrm{i}\cos\theta}{\sin\theta + \mathrm{i}\cos\theta}$ $= \frac{\sin\theta + \mathrm{i}\cos\theta}{\sin\theta}$ $= 1 + \mathrm{i}\cot\theta = \text{RHS}$

(ii)

$$\text{LHS} = 2\arg(z+1)$$
$$= 2\tan^{-1}\left(\frac{\sin\theta}{\cos\theta+1}\right)$$
$$= 2\tan^{-1}\left(\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}\right)$$
$$= 2\tan^{-1}\left(\tan\frac{\theta}{2}\right) = \theta = \text{RHS}$$

Alternative solutions

Triangle formed with these two points and the origin is isosceles.
As $\arg z = \theta$, the obtuse angle in the triangle is $180 - \theta$ and therefore the acute angles are both $\dfrac{\theta}{2}$.

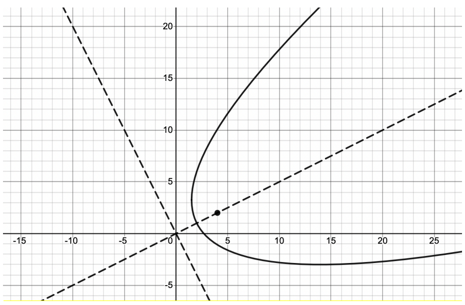


OR

$$2\arg(z=1) - \arg(z) = \arg\left(\frac{(z+1)^2}{z}\right)$$
$$= \arg\left(z + \frac{1}{z} + 2\right)$$
$$= \arg(2\cos\theta + 2) = 0$$

(c)(i)

The locus described in the question is (one definition of) a parabola with directrix $y = -2x$ and focus at P.



(ii)	Distance to point P: $\sqrt{(x-2p)^2+(y-p)^2}$ Distance to point Q: $\sqrt{(x-q)^2+(y+2q)^2}$ $\sqrt{(x-2p)^2+(y-p)^2}=3\sqrt{(x-q)^2+(y+2q)^2}$ $(x-2p)^2+(y-p)^2=9\left[(x-q)^2+(y+2q)^2\right]$ This is a circle, so we need to find its radius r to find its area. $x^2-4px+4p^2+y^2-2py+p^2=9x^2-18qx+9q^2+9y^2+36qy+36q^2$ $8x^2+(4p-18q)x+8y^2+(36q+2p)y+45q^2-5p^2=0$ $x^2+\left(\frac{p}{2}-\frac{9q}{4}\right)x+y^2+\left(\frac{9q}{2}+\frac{p}{4}\right)y+\frac{45}{8}q^2-\frac{5}{8}p^2=0$ By completing the square and focusing just on the constant term we get: $r^2=\frac{5}{8}p^2-\frac{45}{8}q^2+\frac{1}{4}\left(\frac{p}{2}-\frac{9q}{4}\right)^2+\frac{1}{4}\left(\frac{9q}{2}+\frac{p}{4}\right)^2=\frac{45}{64}\left(p^2+q^2\right)$ $A=\frac{45\pi}{64}\left(p^2+q^2\right)$ $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{45\pi}{64}\left(2p\frac{\mathrm{d}p}{\mathrm{d}t}+2q\frac{\mathrm{d}q}{\mathrm{d}t}\right)$ $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{45\pi}{64}\left(2(3)(3)+2(-4)(-2)\right)=\frac{765\pi}{32}\approx 75.1$ OR $A=\frac{45\pi}{64}\left((3+t)^2+(-4-2t)^2\right)=\frac{45\pi}{64}(13t^2+34t+25)$ $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{45\pi}{64}(26t+34)$ Subbing in t = 0 gives: $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{765\pi}{32}$

Scholarship level	Outstanding level
Q4(a) • **1s mark** for any two of the three loci graphed • **1s mark** for the correct area of the region Q4(b)(i) • **1s mark** for Line 2 • **1s mark** for any valid proof Q4(b)(ii) • **1s mark** for Line 3 (half angle formulae used) or diagram showing isosceles triangle, angle θ and $180 - \theta$ labeled Q4(c)(i) • **1s mark** for any two features (directrix, AoS, focus) • **1s mark** for parabola justification Q4(c)(ii) • **1s mark** for using distances to write Line 3 expression *8 marks available* **Max 6 marks awarded**	Q4(b)(ii) • **1o mark** for any valid proof Q4(c)(ii) • **1o mark** for finding the radius of the circle • **1o mark** for finding the correct rate of change required *3 marks available* **Max 2 marks awarded**

Sufficiency Statement

Score 1–4, no award	Score 5–6, Scholarship	Score 7–8, Oustanding Scholarship
Shows understanding of relevant mathematical concepts, and some progress towards solutions to problems.	Application of high-level mathematical knowledge and skills, leading to partial solutions to complex problems.	Application of high-level mathematical knowledge and skills, perception, and insight / convincing communication shown in finding correct solutions to complex problems.

Cut Scores

Scholarship	Outstanding Scholarship
19–27	28–32